Robust Reaction Wheel Attitude Control of Satellites

Sumayya N.,Laila Beebi M., Johnson Y.

Abstract— Attitude control is necessary for the proper functioning of satellite. Actuators used for satellite attitude control include reaction wheel, magnetic torquer, thrusters etc. In this paper design of reaction wheel based on dc motor is considered. Then the performance of satellite is evaluated with reaction wheel. But the system does not settle at the desired attitude. Therefore we first introduce PID controller, then the settling is done at the desired attitude after a large settling time. There after we introduce linear quadratic regulator (LQG) controller, then the system is again become stable and is settled at the desired attitude. The settling time is less as compared to PID controller. Finally robust controller H_∞ is introduced in the system then the settling time is greatly reduced. Non minimum phase response is the drawback of PID controller that is avoided when LQR and H_∞ is used

Index Terms —.Attitude control, attitude error, feedback control, H_∞ controller, LQR controller, non-minimum phase, PID controller, reaction wheel

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1 INTRODUCTION

Attitude is the orientation of a defined spacecraft body coordinate system with respect to a defined external frame. Attitude determination includes real-time or post-facto knowledge, within a given tolerance, of the spacecraft attitude [2, 3]. Maintenance of a desired, specified attitude within a given tolerance is attitude control. Attitude control is necessary for the proper functioning of satellite. Low frequency spacecraft misalignment is termed as attitude error, usually the intended topic of attitude control. High frequency spacecraft misalignment is termed as attitude jitter, usually ignored by attitude determination and control, reduced by good design or fine pointing or optical control

Active control systems directly sense spacecraft attitude and supply a torque command to alter it. This is the basic concept of feedback control [4, 5].This is the basic concept of feedback control and is called active attitude control. Magnetic torque is a simple and reliable technology for the attitude acquisition and regulation of small satellites in low earth orbit by interacting with the magnetic field of the Earth through their magnetic dipoles [1].Passive control techniques take advantage of basic physical principles and/or naturally occurring forces by designing the space craft so as to enhance the effect of one force, while reducing the effect of others.

Reaction wheel is the most common actuator that provides fast and continuous feedback control and having moving parts. It has internal torque only, external still required for momentum dumping. It has relatively high power, weight and cost. Control logic is simple for independent axes. Reaction wheels are simple disks (rotors) that can run by an electric motor. When the motor applies a torque to speed up or slow down the rotor, it produces a reacting torque on the body of satellite. Since satellite is essentially a closed loop system, the total angular momentum of the satellite body plus the reaction wheel is constant. Thus any change in angular momentum of reaction wheel result in an equal and opposite change of the angular momentum of satellite body.

One reaction wheel can affect the satellite's momentum along one axis. To control the satellite along all three axes, at least three reaction wheels are required, use four wheels for redundancy. If external torque exists, wheels will angularly accelerate to counteract this torque. In reaction wheels errors are produced in the output of reaction wheels due to the change in value of resistance caused by the heating effect produced by the flow of current. Also errors are produced due to external disturbances. In this paper mathematical modelling of reaction wheels is carried out and design and implementation of controllers are done to reduce the error due to variation in resistance caused by the flow of currents. The aim is to produce better output for reaction wheels and thereby proper controlling of attitude.

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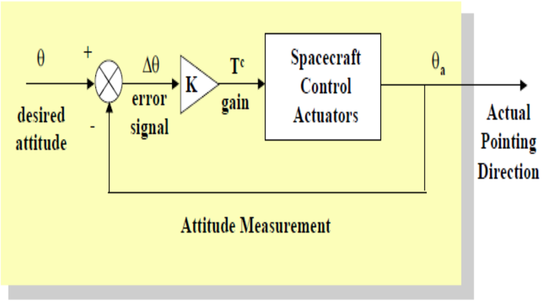

Fig.1: Active attitude control

Thrusters can be used to control attitude but at the cost of consuming fuel. Use consumables such as cold gas or hydrazine. They are fast and powerful. Redundancy usually required makes the system more complex and expensive

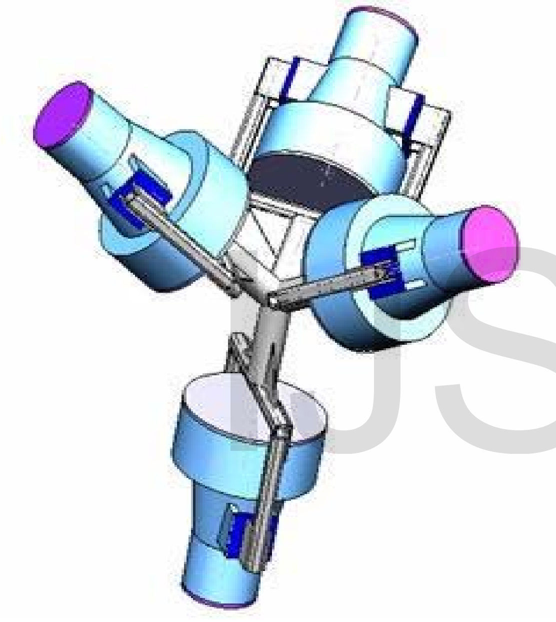

Fig.2: Tetrahedral configuration of reaction wheel

2 MODELLING

Reaction wheels are driven by brushless dc motor.

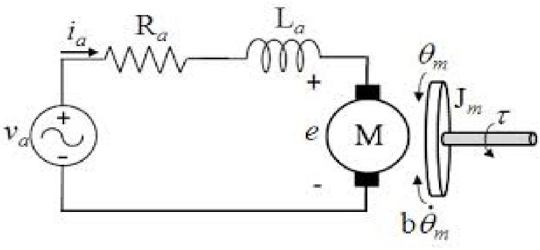

Fig.3: Equivalent circuit of DC motor

By Kirchhoff's voltage law we can write, [6]

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a \qquad (1)$$

The torque is proportional to armature current

$$T \, \alpha i_a$$

$$T = K_t \, i_a \qquad (2)$$

The differential equation governing the mechanical system of the motor is given by

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \qquad (3)$$

The back emf of dc machine is proportional to speed (angular velocity) of the shaft

$$e_b \alpha \frac{d\theta}{dt}$$

$$e_b = K_b \frac{d\theta}{dt} \qquad (4)$$

Taking Laplace transform of equations (1),(2),(3)&(4), we get

$$R_a I_a(s) + L_a s I_a(s) + E_b(s) = V_a(s) \qquad (5)$$

$$T(s) = K_t \, I_a(s) \quad (6)$$

$$JS^2\theta(s) + Bs\theta(s) = T(s) \qquad (7)$$

$$E_b(s) = K_b s\theta(s) \qquad (8)$$

On equating (6) & (7), we get

$$I_a(s) = \frac{\theta(S)\,(JS^2 + BS)}{K_t} \qquad (9)$$

Substituting (8) & (9) in (5) and rearranging them we got the transfer function

$$\frac{\theta(S)}{V_a(s)} = \frac{K_t}{(JS^2 + BS)(R_a + L_a s) + K_b K_t S} \qquad (10)$$

From (4) we get

$$e_b = K_b \, W_m \qquad (11)$$

Where $\frac{d\theta}{dt} = W_m$, angular velocity of rotor shaft

The work load of the DC motor consists of a reaction wheel mounted on the rotor shaft, resulting in the work load torque [7, 8].

$$T_L = I_W \, \dot{W}_W \qquad (12)$$

I_W = Moment of inertia of the wheel

W_W = Angular velocity of the wheel

The rotor shaft of the motor has the same angular velocity and acceleration with the reaction wheel, there fore

$$W_m = W_W \& \dot{W}_m = \dot{W}_W \qquad (13)$$

The moment balance equation of rotor shaft is

$$I_m \dot{W}_m = T - T_L \qquad (14)$$

Substitute (12) & (13) in (14), we get

$$(I_m + I_W)\,\dot{W}_m = T \qquad (15)$$

Assume that $I_W \gg I_m$, the reaction wheel is added to the system to increase the moment of inertia then (15) becomes

$$T = I_W \dot{W}_m \qquad (16)$$

From this torque command to the motor, angular velocity of rotor shaft is calculated, then the voltage that needs to be applied to motor is calculated.

3 BLOCK DIAGRAM

From the electrical and mechanical dynamics of the reaction wheel we get

$$L_a \frac{di_a}{dt} = V - i_a R_a - K_b W_m \qquad (17)$$

$$J\frac{dW_m}{dt} = K_t\, i_a - BW_m \qquad (18)$$

We can draw the block diagram of each equation, joining that two block diagrams we get the complete block diagram of entire reaction wheel [12].

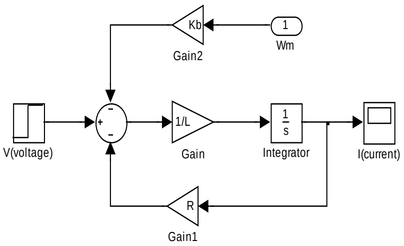

Fig.4: Block diagram of electrical dynamics

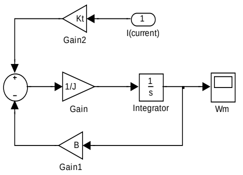

Fig.5: Block diagram of mechanical dynamics

Complete block diagram of entire reaction wheel is shown in fig.6. And the responses are obtained from the block diagram. In block diagram we can represent only one reaction wheel

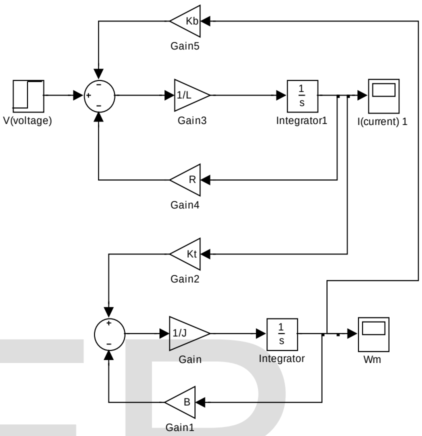

Fig.6. Complete block diagram of entire reaction wheel

By putting the values of each term in the block diagram we got the responses of reaction wheel. It includes angular velocity, current and their derivatives. From the response it is clear that the system does not settle at the desired attitude.

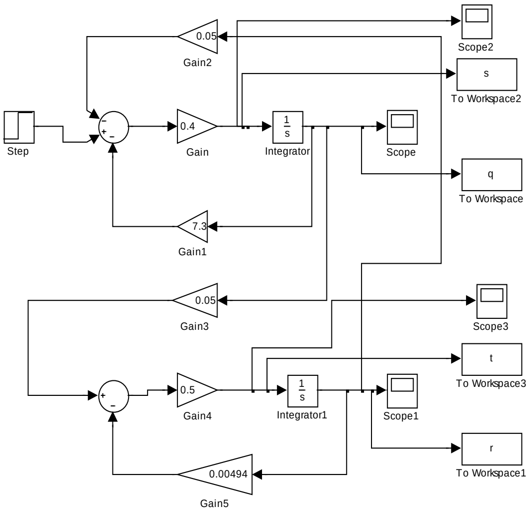

Fig.7. Complete block diagram of entire reaction wheel with values

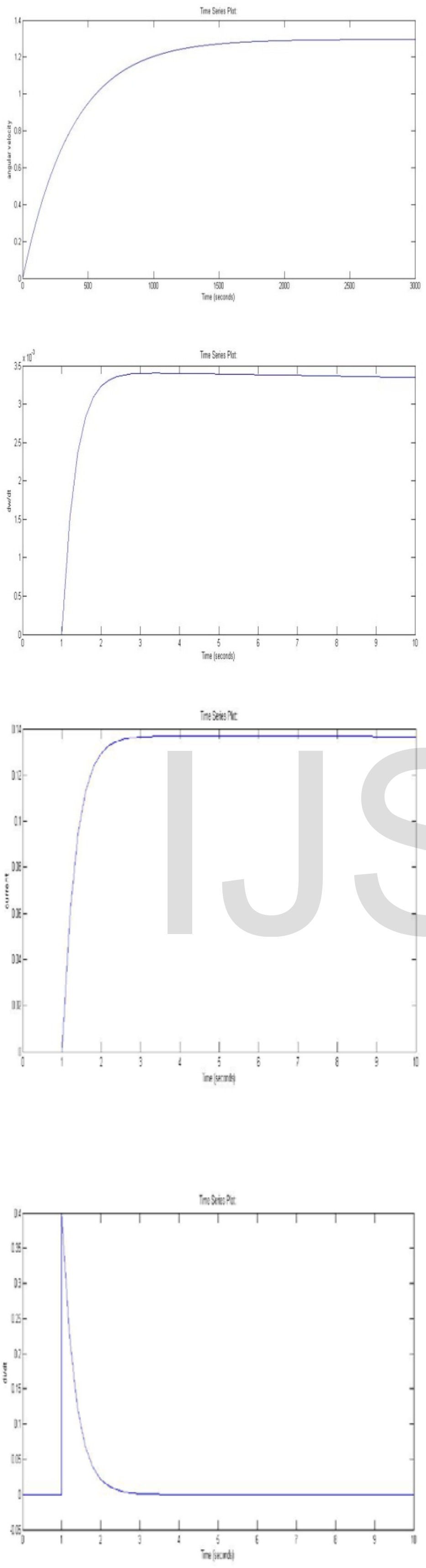

Fig.8. Responses of reaction wheel

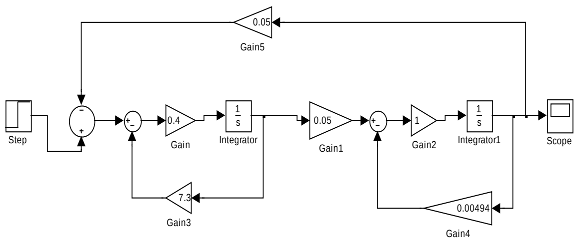

Fig.9. Reaction wheel in subsystem form

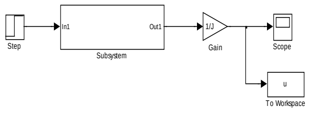

Fig.10. Reaction wheel with satellite [11]

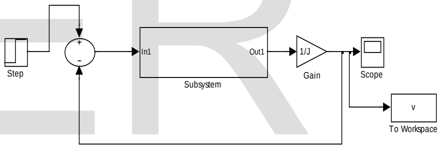

Fig. 11.Rection wheel with satellite closed loop structure

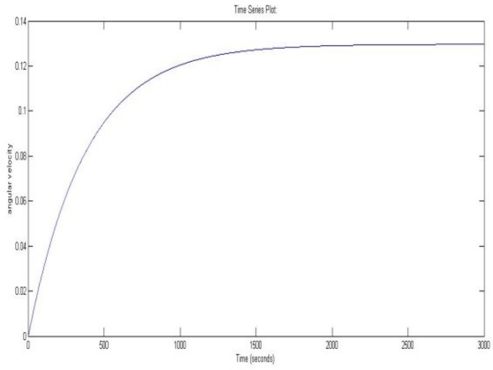

Fig.12. Reaction wheel with satellite open loop response (angular velocity)

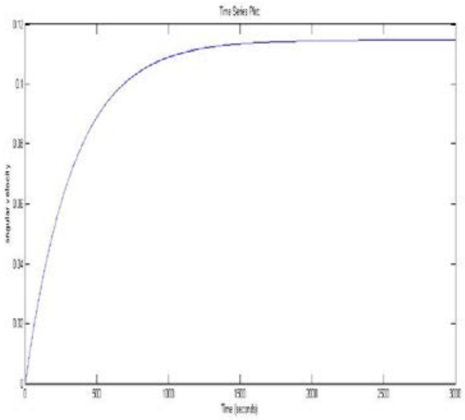

Fig.13. Reaction wheel with satellite closed loop response (angular velocity)

The closed loop system output does not become stable at the desired attitude also the settling time is very large. So there is a need of controller.

4. PID CONTROLLER

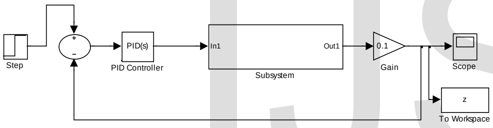

Fig.14. Reaction wheel with satellite & PID controller

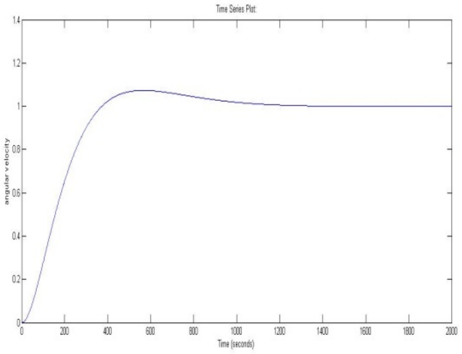

Fig.15. Response of reaction wheel with satellite & PID controller from block (angular velocity)

With the introduction of PID controller in the system, the response shows that the system is become stable and is settled at the desired attitude. But the settling time is large. Therefore to improve the performance, suitable controller is necessary. Also PID controller does not maintain the desired attitude initially under disturbance conditions. Over shoots are present in the response

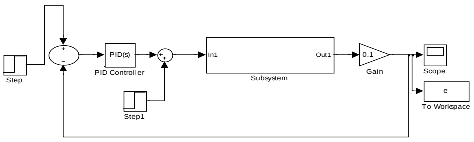

Fig.16. System with PID controller and disturbance

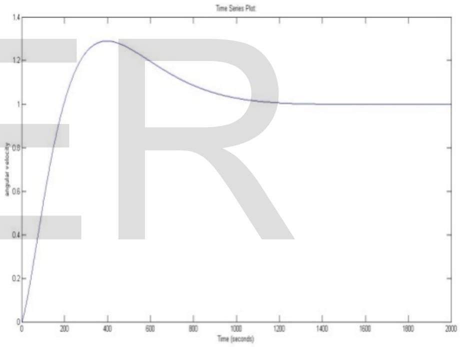

Fig.17. Response of the disturbed system with PID controller (angular velocity)

Assessment of expected disturbance is an essential part of space craft attitude control design. Here we gave the disturbance in the form of step input. Normally the disturbances in space include gravity gradient (tidal force due to gravitational field variation for long, extended bodies), magnetic torques (induced by residual magnetic moment, here space craft is modelled as a magnetic dipole only within magnetosphere), disturbance due to solar effect (can be compensated with differential reflectivity of reaction wheel, mass expulsion (torques induced by leaks or jettisoned objects) and the last one, the internal disturbance. The system includes on board equipments, machinery, wheels, cryocoolers, pumps etc. This does not produce any net effect but internal momentum exchanges affect attitude.

5 STATE SPACE REPRESENTATION

The electrical and mechanical dynamics equations can be combined to model the motion of three reaction wheels [9, 10].

$$\frac{di_a}{dt} = \frac{v}{L_a} - i_a\frac{R_a}{L_a} - \frac{K_b}{L_a}w_m \qquad (19)$$

$$\frac{dw_m}{dt} = \frac{K_t}{J}i_a - \frac{B}{J}w_m \qquad (20)$$

(19) & (20) can be modified as

$$i'_a = \frac{v}{L_a} - i_a\frac{R_a}{L_a} - \frac{K_b}{L_a}w_m \qquad (21)$$

$$w'_m = \frac{K_t}{J}i_a - \frac{B}{J}w_m \qquad (22)$$

Combined model of the electrical and mechanical dynamics of the reaction wheel are

$$\begin{bmatrix} w'_x \\ w'_y \\ w'_z \\ i'_x \\ i'_y \\ i'_z \end{bmatrix} = \begin{bmatrix} -\frac{B}{J} & 0 & 0 & \frac{K_t}{J} & 0 & 0 \\ 0 & -\frac{B}{J} & 0 & 0 & \frac{K_t}{J} & 0 \\ 0 & 0 & -\frac{B}{J} & 0 & 0 & \frac{K_t}{J} \\ \frac{K_b}{L_a} & 0 & 0 & -\frac{R_a}{L_a} & 0 & 0 \\ 0 & \frac{K_b}{L_a} & 0 & 0 & -\frac{R_a}{L_a} & 0 \\ 0 & 0 & \frac{K_b}{L_a} & 0 & 0 & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \\ i_x \\ i_y \\ i_z \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{L_a} & 0 & 0 \\ 0 & \frac{1}{L_a} & 0 \\ 0 & 0 & \frac{1}{L_a} \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

$$\begin{bmatrix} w_x \\ w_y \\ w_z \\ i_x \\ i_y \\ i_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \\ i_x \\ i_y \\ i_z \end{bmatrix} + 0$$

In block diagram representation we can consider only one reaction wheel but in state space representation we can consider the reaction wheels in three directions. Also by MATLAB programming we got the transfer function for angular velocity and current in all the three directions. DC motors used for three reaction wheels are of same specifications. Then the transfer function of angular velocity is same in all the three directions, also the transfer function of current is same.

From the state space representation by using MATLAB programming we got transfer function for both angular velocity and current. The transfer function obtained for reaction wheel is

W = 0.01/(s^2 + 2.922 s + 0.007712)

I = 0.4 s + 0.000988/(s^2 + 2.922 s + 0.007712)

Responses for angular velocity and current obtained from transfer function are same as the responses that are obtained from the block diagram. By combining reaction wheel and satellite we got the transfer function for both open loop and closed loop. The responses obtained for this is also same as the responses that are obtained from the block diagram. But the responses of the system with PID controller show differences when comparing the responses of block diagram and MATLAB programme.

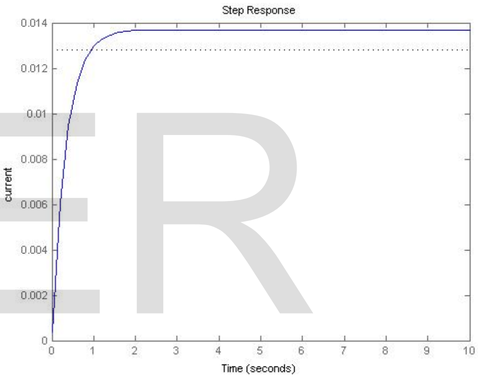

Fig.18. Reaction wheel with satellite open loop response (current)

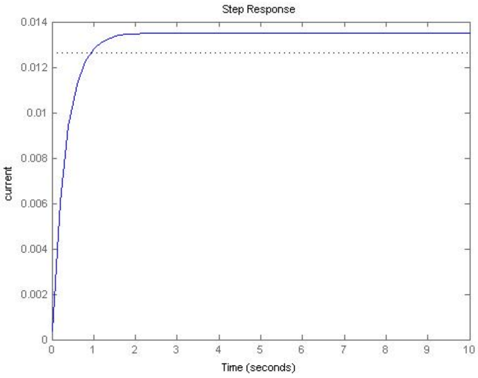

Fig.19. Reaction wheel with satellite closed loop response (current)

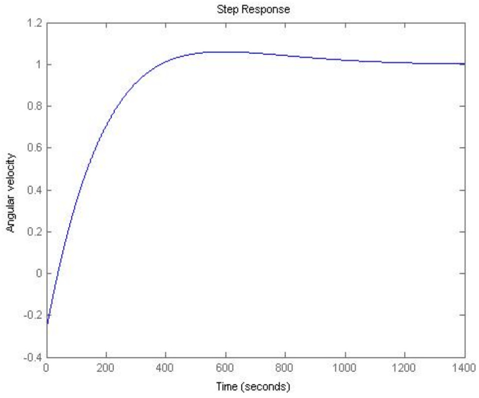

Fig.20. Closed loop response of reaction wheel with satellite & PID controller from programme (angular velocity)

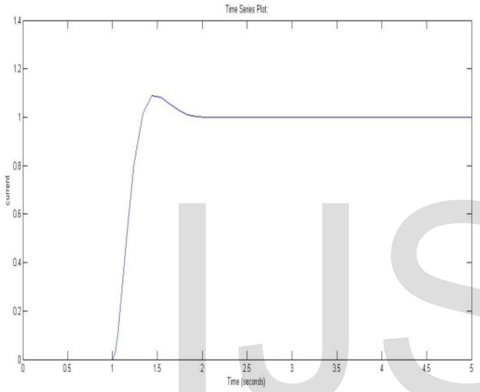

Fig.21. Response of reaction wheel with satellite & PID controller from block (current)

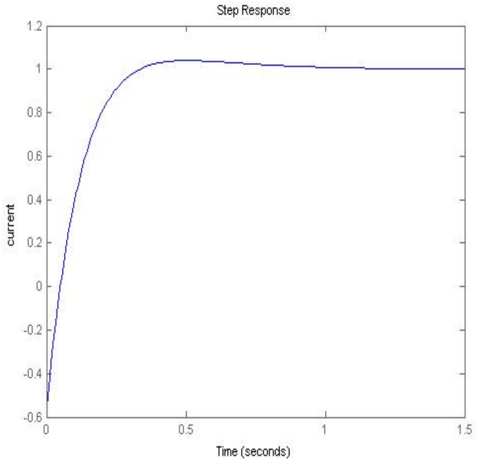

Fig.22. Response of reaction wheel with satellite & PID controller from programme (current)

From the response we can see that the response that goes in to the negative direction and there after it increases. Such a system is known as non minimum phase system [13, 14]. This is due to the presence of right half plane zero in the transfer function. When the openloop system has a right-half plane (non minimum phase) zero,the step response spends part of its time going in the negative direction. This generally known as a non minimum phase response or an inverse response. This inverse response always exists when the closed-loop system has a right half plane zero. The system's step response will exhibit undershoot, taking on negative values. Non minimum phase response is a drawback of PID controller. PID controller does not have an ability to maintain stability in real disturbed conditions. So a robust controller is necessary to avoid these drawbacks.

6 LQR CONTROLLER

Consider the state space system

$\dot{x} = Ax + Bu$

$y = Cx$

Performance criterion

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T Ru(t)]dt \qquad (19)$$

where Q is non negative definite and R is positive definite. Then the optimal control minimising (19) is given by the linear state feedback law

$u(t) = -Kx(t)$, with $K = R^{-1}B^TP$

and where P is the unique positive definite solution to the matrix Algebraic Riccati Equation (ARE).

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \qquad (20)$$

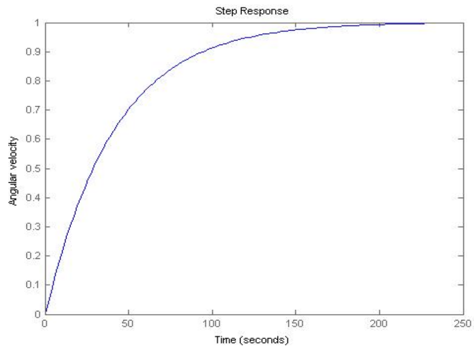

Fig.23. Response of the system with LQR controller (angular velocity)

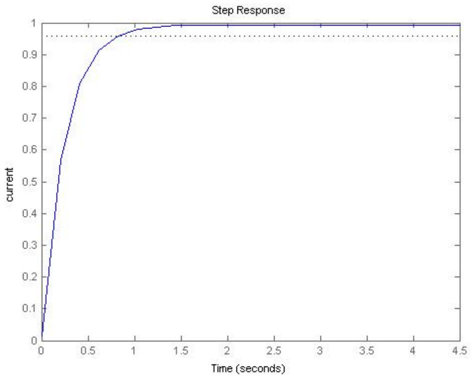

Fig.24. Response of the system with LQR controller (current)

From the response it is clear that LQR provides better result as compared to PID controller. Settling time is greatly reduced and the system that settle at the desired attitude. By using LQR controller we can control both current and angular velocity.

7 H_∞ CONTROLLER

H_∞ controller possess the ability to maintain stability in real disturbed conditions [14,15].

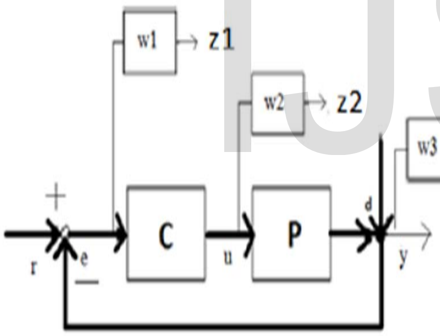

Fig.25.General block diagram of H_∞ controller

By combining the transfer functions of the plant and some tuning weights an augmented matrix is formed. Some preliminary conditions are required to be satisfied for their selection to the given situation. The preliminary condition to be satisfied is that the infinity norm of the product of weights assigned and thesensitivity(S) and the complementary sensitivity(T) should be less than one. Three weights are needed to be tuned for performance and stability achievement. For achieving good disturbance rejection, the weight W1 should be properly selected. If stability margin is the main concern, W3 should be tuned.

On applying H_∞ controller on reaction wheel including satellite we got better results.

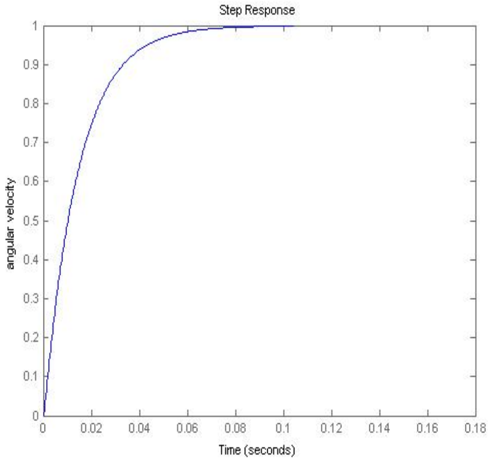

Fig.26. Response of the system with H infinity controller (angular velocity)

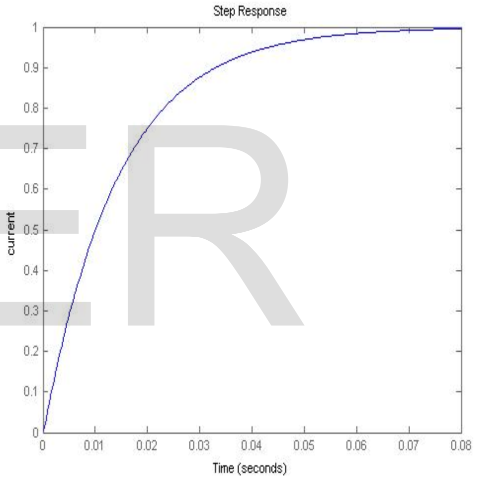

Fig. 27. Response of the system with H infinity controller (current)

From this response it is clear that H_∞ controller gives very good responses and performance is improved. The settling time is greatly reduced. The system is settled at the desired attitude with in 0.08 seconds. Settling time is very low as compared with PID and LQR controller. For applying H_∞ controller certain conditions should be satisfied, for that purpose the sensitivity and complementary sensitivity graphs should be drawn. The sensitivity and complementary sensitivity graphs of angular velocity and current are shown below.

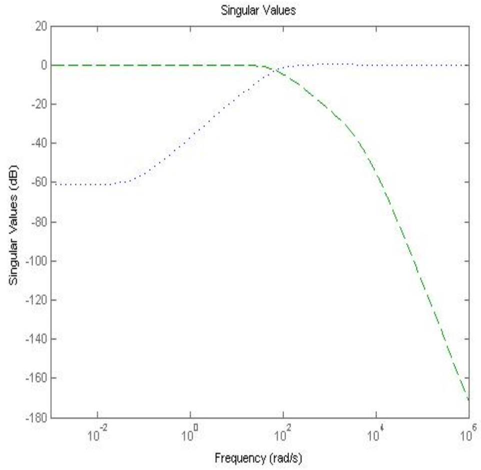

Fig.28. Sensitivity and complementary sensitivity graph (angular velocity)

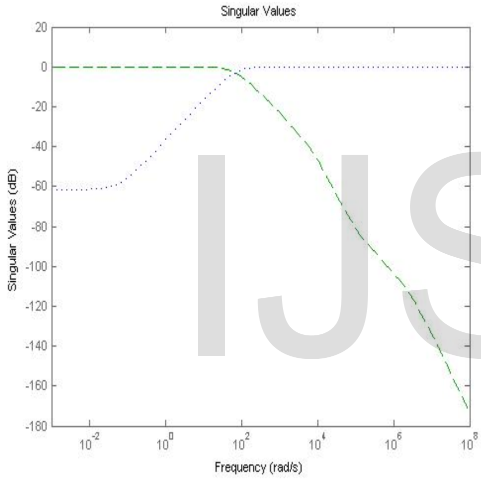

Fig.29. Sensitivity and complementary sensitivity graph (current)

8 CONCLUSION

Reaction wheel is a most common actuator used for the attitude control of satellites. It is a stable system but it does not settle at the desired attitude in the absence of a controller and also in the response settling time is very large. On applying PID controller in the system better results are obtained, the system is settled at the desired attitude but also in the response, settling time is large. Non minimum phase response is a drawback of PID controller. PID controller does not have an ability to maintain stability in real disturbed conditions. On applying LQR controller the system is settled at the desired attitude and settling time in the response is reduced as compared to PID controller. H_{∞} controller provides better results and settling time is reduced to a greater extent as compared to PID and LQR controller. The drawback of PID controller that is its non minimum phase response, that is avoided with the use of LQR and H_{∞} controller. In PID controller the settling time is about 980 seconds and in LQR controller the settling time is about 160 seconds but in H_{∞} controller the settling time is about 0.08 seconds (as in the case of angular velocity) that means H_{∞} provides excellent results. H_{∞} controller and LQR controller possess the ability to maintain stability in real disturbed conditions.

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